**LOGICA MATEMATICA ESERCIZI SVOLTI**

1. **DETERMINARE LA TAVOLA DI VERITA’ DELLA SEGUENTE PROPOSIZIONE MOLECOLARE**

$$(p∧\overbar{q})\rightarrow \left(\overbar{p}∨q\right)$$

**Primo passaggio: inserimento dei valori logici delle proposizioni atomiche** $p$ **e** $q$

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| $$p$$ | $$q$$ | $$\overbar{p}$$ | $$\overbar{q}$$ | $$p∧\overbar{q}$$ | $$\overbar{p}∨q$$ | $(p∧\overbar{q})\rightarrow \left(\overbar{p}∨q\right)$ |
| **V** | **V** | **F** | **F** | **F** | **V** | **V** |
| **V** | **F** | **F** | **V** | **V** | **F** | **F** |
| **F** | **V** | **V** | **F** | **F** | **V** | **V** |
| **F** | **F** | **V** | **V** | **F** | **V** | **V** |

**Secondo passaggio: negazione della 1 e 2 colonna**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| $$p$$ | $$q$$ | $$\overbar{p}$$ | $$\overbar{q}$$ | $$p∧\overbar{q}$$ | $$\overbar{p}∨q$$ | $(p∧\overbar{q})\rightarrow \left(\overbar{p}∨q\right)$ |
| **V** | **V** | **F** | **F** | **F** | **V** | **V** |
| **V** | **F** | **F** | **V** | **V** | **F** | **F** |
| **F** | **V** | **V** | **F** | **F** | **V** | **V** |
| **F** | **F** | **V** | **V** | **F** | **V** | **V** |

**Terzo passaggio: congiunzione tra la 1 e la 4 colonna**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| $$p$$ | $$q$$ | $$\overbar{p}$$ | $$\overbar{q}$$ | $$p∧\overbar{q}$$ | $$\overbar{p}∨q$$ | $(p∧\overbar{q})\rightarrow \left(\overbar{p}∨q\right)$ |
| **V** | **V** | **F** | **F** | **F** | **V** | **V** |
| **V** | **F** | **F** | **V** | **V** | **F** | **F** |
| **F** | **V** | **V** | **F** | **F** | **V** | **V** |
| **F** | **F** | **V** | **V** | **F** | **V** | **V** |

**Quarto passaggio: disgiunzione inclusiva tra la 3 e la 2 colonna**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| $$p$$ | $$q$$ | $$\overbar{p}$$ | $$\overbar{q}$$ | $$p∧\overbar{q}$$ | $$\overbar{p}∨q$$ | $(p∧\overbar{q})\rightarrow \left(\overbar{p}∨q\right)$ |
| **V** | **V** | **F** | **F** | **F** | **V** | **V** |
| **V** | **F** | **F** | **V** | **V** | **F** | **F** |
| **F** | **V** | **V** | **F** | **F** | **V** | **V** |
| **F** | **F** | **V** | **V** | **F** | **V** | **V** |

**Quinto passaggio: implicazione tra la 5 e la 6 colonna**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| $$p$$ | $$q$$ | $$\overbar{p}$$ | $$\overbar{q}$$ | $$p∧\overbar{q}$$ | $$\overbar{p}∨q$$ | $ (p∧\overbar{q})\rightarrow \left(\overbar{p}∨q\right)$ |
| **V** | **V** | **F** | **F** | **F** | **V** | **V** |
| **V** | **F** | **F** | **V** | **V** | **F** | **F** |
| **F** | **V** | **V** | **F** | **F** | **V** | **V** |
| **F** | **F** | **V** | **V** | **F** | **V** | **V** |

1. $ $**DETERMINARE LA TAVOLA DI VERITA’ DELLA SEGUENTE PROPOSIZIONE MOLECOLARE**

$$(p∧q)\leftrightarrow \left(q∧p\right)$$

**Primo passaggio: inserimento dei valori logici delle proposizioni atomiche** $p$ **e** $q$

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$p$$ | $$q$$ | $$p∧q$$ | $$q∧p$$ | $(p∧q)\leftrightarrow \left(q∧p\right)$ |
| **V** | **V** | **V** | **V** | **V** |
| **V** | **F** | **F** | **F** | **V** |
| **F** | **V** | **F** | **F** | **V** |
| **F** | **F** | **F** | **F** | **V** |

**Secondo passaggio: congiunzione tra la 1 e la 2 colonna**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$p$$ | $$q$$ | $$p∧q$$ | $$q∧p$$ | $(p∧q)\leftrightarrow \left(q∧p\right)$ |
| **V** | **V** | **V** | **V** | **V** |
| **V** | **F** | **F** | **F** | **V** |
| **F** | **V** | **F** | **F** | **V** |
| **F** | **F** | **F** | **F** | **V** |

**Terzo passaggio: congiunzione tra la 2 e la 1 colonna**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$p$$ | $$q$$ | $$p∧q$$ | $$q∧p$$ | $(p∧q)\leftrightarrow \left(q∧p\right)$ |
| **V** | **V** | **V** | **V** | **V** |
| **V** | **F** | **F** | **F** | **V** |
| **F** | **V** | **F** | **F** | **V** |
| **F** | **F** | **F** | **F** | **V** |

**Quarto passaggio: coimplicazione tra la 3 e la 4 colonna**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$p$$ | $$q$$ | $$p∧q$$ | $$q∧p$$ | $ (p∧q)\leftrightarrow \left(q∧p\right)$ |
| **V** | **V** | **V** | **V** | **V** |
| **V** | **F** | **F** | **F** | **V** |
| **F** | **V** | **F** | **F** | **V** |
| **F** | **F** | **F** | **F** | **V** |

**L’enunciato composto è una tautologia perché è una proposizione molecolare sempre vera, infatti esprime la legge commutativa della congiunzione.**

1. **DETERMINARE LA TAVOLA DI VERITA’ DELLA SEGUENTE PROPOSIZIONE MOLECOLARE**

$$(\overbar{p∨q})\leftrightarrow \left(\overbar{p}∧\overbar{q}\right)$$

**Primo passaggio: inserimento dei valori logici delle proposizioni atomiche** $p$ **e** $q$

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| $$p$$ | $$q$$ | $$\overbar{p}$$ | $$\overbar{q}$$ | $$p∨q$$ | $$\overbar{p∨q}$$ | $$\overbar{p}∧\overbar{q}$$ | $ (\overbar{p∨q})\leftrightarrow \left(\overbar{p}∧\overbar{q}\right)$ |
| **V** | **V** | **F** | **F** | **V** | **F** | **F** | **V** |
| **V** | **F** | **F** | **V** | **V** | **F** | **F** | **V** |
| **F** | **V** | **V** | **F** | **V** | **F** | **F** | **V** |
| **F** | **F** | **V** | **V** | **F** | **V** | **V** | **V** |

**Secondo passaggio: negazione della 1 e 2 colonna**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| $$p$$ | $$q$$ | $$\overbar{p}$$ | $$\overbar{q}$$ | $$p∨q$$ | $$\overbar{p∨q}$$ | $$\overbar{p}∧\overbar{q}$$ | $ (\overbar{p∨q})\leftrightarrow \left(\overbar{p}∧\overbar{q}\right)$ |
| **V** | **V** | **F** | **F** | **V** | **F** | **F** | **V** |
| **V** | **F** | **F** | **V** | **V** | **F** | **F** | **V** |
| **F** | **V** | **V** | **F** | **V** | **F** | **F** | **V** |
| **F** | **F** | **V** | **V** | **F** | **V** | **V** | **V** |

**Terzo passaggio: disgiunzione inclusiva tra la 1 e la 2 colonna**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| $$p$$ | $$q$$ | $$\overbar{p}$$ | $$\overbar{q}$$ | $$p∨q$$ | $$\overbar{p∨q}$$ | $$\overbar{p}∧\overbar{q}$$ | $ (\overbar{p∨q})\leftrightarrow \left(\overbar{p}∧\overbar{q}\right)$ |
| **V** | **V** | **F** | **F** | **V** | **F** | **F** | **V** |
| **V** | **F** | **F** | **V** | **V** | **F** | **F** | **V** |
| **F** | **V** | **V** | **F** | **V** | **F** | **F** | **V** |
| **F** | **F** | **V** | **V** | **F** | **V** | **V** | **V** |

**Quarto passaggio: negazione della 5 colonna**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| $$p$$ | $$q$$ | $$\overbar{p}$$ | $$\overbar{q}$$ | $$p∨q$$ | $$\overbar{p∨q}$$ | $$\overbar{p}∧\overbar{q}$$ | $ (\overbar{p∨q})\leftrightarrow \left(\overbar{p}∧\overbar{q}\right)$ |
| **V** | **V** | **F** | **F** | **V** | **F** | **F** | **V** |
| **V** | **F** | **F** | **V** | **V** | **F** | **F** | **V** |
| **F** | **V** | **V** | **F** | **V** | **F** | **F** | **V** |
| **F** | **F** | **V** | **V** | **F** | **V** | **V** | **V** |

**Quinto passaggio: congiunzione tra la 3 e la 4 colonna**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| $$p$$ | $$q$$ | $$\overbar{p}$$ | $$\overbar{q}$$ | $$p∨q$$ | $$\overbar{p∨q}$$ | $$\overbar{p}∧\overbar{q}$$ | $ (\overbar{p∨q})\leftrightarrow \left(\overbar{p}∧\overbar{q}\right)$ |
| **V** | **V** | **F** | **F** | **V** | **F** | **F** | **V** |
| **V** | **F** | **F** | **V** | **V** | **F** | **F** | **V** |
| **F** | **V** | **V** | **F** | **V** | **F** | **F** | **V** |
| **F** | **F** | **V** | **V** | **F** | **V** | **V** | **V** |

**Sesto passaggio: coimplicazione tra la 6 e la 7 colonna**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| $$p$$ | $$q$$ | $$\overbar{p}$$ | $$\overbar{q}$$ | $$p∨q$$ | $$\overbar{p∨q}$$ | $$\overbar{p}∧\overbar{q}$$ | $ (\overbar{p∨q})\leftrightarrow \left(\overbar{p}∧\overbar{q}\right)$ |
| **V** | **V** | **F** | **F** | **V** | **F** | **F** | **V** |
| **V** | **F** | **F** | **V** | **V** | **F** | **F** | **V** |
| **F** | **V** | **V** | **F** | **V** | **F** | **F** | **V** |
| **F** | **F** | **V** | **V** | **F** | **V** | **V** | **V** |

**PRIMA LEGGE DI DE MORGAN**

1. **DETERMINARE LA TAVOLA DI VERITA’ DELLA SEGUENTE PROPOSIZIONE MOLECOLARE**

$$(\overbar{p∨q})\leftrightarrow \left(\overbar{p}∧\overbar{q}\right)$$

**Primo passaggio: inserimento dei valori logici delle proposizioni atomiche** $p$ **e** $q$

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| $$p$$ | $$q$$ | $$\overbar{p}$$ | $$\overbar{q}$$ | $$p∧q$$ | $$\overbar{p∧q}$$ | $$\overbar{p}∨\overbar{q}$$ | $ (\overbar{p∧q})\leftrightarrow \left(\overbar{p}∨\overbar{q}\right)$ |
| **V** | **V** | **F** | **F** | **V** | **F** | **F** | **V** |
| **V** | **F** | **F** | **V** | **V** | **F** | **F** | **V** |
| **F** | **V** | **V** | **F** | **V** | **F** | **F** | **V** |
| **F** | **F** | **V** | **V** | **F** | **V** | **V** | **V** |

**Secondo passaggio: negazione della 1 e 2 colonna**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| $$p$$ | $$q$$ | $$\overbar{p}$$ | $$\overbar{q}$$ | $$p∧q$$ | $$\overbar{p∧q}$$ | $$\overbar{p}∨\overbar{q}$$ | $ (\overbar{p∧q})\leftrightarrow \left(\overbar{p}∨\overbar{q}\right)$ |
| **V** | **V** | **F** | **F** | **V** | **F** | **F** | **V** |
| **V** | **F** | **F** | **V** | **V** | **F** | **F** | **V** |
| **F** | **V** | **V** | **F** | **V** | **F** | **F** | **V** |
| **F** | **F** | **V** | **V** | **F** | **V** | **V** | **V** |

**Terzo passaggio: congiunzione tra la 1 e la 2 colonna**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| $$p$$ | $$q$$ | $$\overbar{p}$$ | $$\overbar{q}$$ | $$p∧q$$ | $$\overbar{p∧q}$$ | $$\overbar{p}∨\overbar{q}$$ | $ (\overbar{p∧q})\leftrightarrow \left(\overbar{p}∨\overbar{q}\right)$ |
| **V** | **V** | **F** | **F** | **V** | **F** | **F** | **V** |
| **V** | **F** | **F** | **V** | **F** | **F** | **F** | **V** |
| **F** | **V** | **V** | **F** | **F** | **F** | **F** | **V** |
| **F** | **F** | **V** | **V** | **F** | **V** | **V** | **V** |

**Quarto passaggio: negazione della 5 colonna**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| $$p$$ | $$q$$ | $$\overbar{p}$$ | $$\overbar{q}$$ | $$p∧q$$ | $$\overbar{p∧q}$$ | $$\overbar{p}∨\overbar{q}$$ | $ (\overbar{p∧q})\leftrightarrow \left(\overbar{p}∨\overbar{q}\right)$ |
| **V** | **V** | **F** | **F** | **V** | **F** | **F** | **V** |
| **V** | **F** | **F** | **V** | **F** | **V** | **F** | **V** |
| **F** | **V** | **V** | **F** | **F** | **V** | **F** | **V** |
| **F** | **F** | **V** | **V** | **F** | **V** | **V** | **V** |

**Quinto passaggio: disgiunzione inclusiva tra la 3 e la 4 colonna**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| $$p$$ | $$q$$ | $$\overbar{p}$$ | $$\overbar{q}$$ | $$p∧q$$ | $$\overbar{p∧q}$$ | $$\overbar{p}∨\overbar{q}$$ | $ (\overbar{p∧q})\leftrightarrow \left(\overbar{p}∨\overbar{q}\right)$ |
| **V** | **V** | **F** | **F** | **V** | **F** | **F** | **V** |
| **V** | **F** | **F** | **V** | **F** | **V** | **V** | **V** |
| **F** | **V** | **V** | **F** | **F** | **V** | **V** | **V** |
| **F** | **F** | **V** | **V** | **F** | **V** | **V** | **V** |

**Sesto passaggio: coimplicazione tra la 6 e la 7 colonna**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| $$p$$ | $$q$$ | $$\overbar{p}$$ | $$\overbar{q}$$ | $$p∧q$$ | $$\overbar{p∧q}$$ | $$\overbar{p}∨\overbar{q}$$ | $ (\overbar{p∧q})\leftrightarrow \left(\overbar{p}∨\overbar{q}\right)$ |
| **V** | **V** | **F** | **F** | **V** | **F** | **F** | **V** |
| **V** | **F** | **F** | **V** | **F** | **V** | **V** | **V** |
| **F** | **V** | **V** | **F** | **F** | **V** | **V** | **V** |
| **F** | **F** | **V** | **V** | **F** | **V** | **V** | **V** |

**SECONDA LEGGE DI DE MORGAN**